

Chapter 17 – Parameterized Curves & Vector Fields

17.1 Parameterized curves

Parameterize a linear path between two points

Parameterize a graph $y = f(x)$

Parameterize a circle (with either clockwise or counterclockwise motion)

(Note: In some problems we are concerned with where a particle is at a given time on a parameterized curve. For other problems, we are only concerned with the shape of the curve - there are many ways to parameterize the same shape.)

17.2 Motion, velocity and acceleration

Using position vectors to write parameterized curves as vector functions $\mathbf{r}(t)$

Velocity vector: $\mathbf{v} = \mathbf{r}'(t)$

Acceleration vector: $\mathbf{a} = \mathbf{r}''(t)$

Speed = $\|\mathbf{v}\|$

Finding the length of a curve: Integrate the *speed* over the time interval.

17.3 Vector Fields

Velocity, force, and gradient fields

17.4 The flow of a vector field.

A flow line of a vector field can be visualized as follows: Pretend that the vector field represents the movement of a fluid. A flow line is the path that a cork would take if dropped into the fluid.

Be able to set up the system of differential equations defining a flow line.

(You should be able to solve such systems when the flow line is a *linear* path.)

The flow of a vector field is the set of all of its flow lines.

Chapter 18 – Line Integrals

18.1 The idea of a line integral

Be able to determine the *sign* of a line integral graphically

Limit definition of a line integral

It is often helpful to refer to the specific example of *work* as a line integral

Circulation = line integral over a closed curve C

Properties of line integrals

18.2 Computing line integrals over parameterized curves

$$\text{Method of Parameterization: } \int_c \vec{F} \cdot d\vec{r} = \int_a^b \vec{F}(\vec{r}(t)) \cdot \vec{r}'(t) dt$$

Independence of parameterization (only the shape and orientation of curve matter)

18.3 Gradient fields and path-independent fields

Fundamental theorem of calculus for line integrals (conservative fields only)

Curl of 2D & 3D fields.

Curl Test to determine if a field is conservative

Finding a potential function (a.k.a anti-gradient function)

18.4 Path-dependent vector fields and Green's Theorem

Green's theorem provides allows us (under certain conditions) to convert line integrals to double integrals, and vice versa.

Green's theorem also allows you to find the *area* of a region by integrating around its border.

Overarching idea: There are multiple ways of evaluating line integrals: Method of Parameterization, FTC for Line Integrals and Green's Theorem. Be able to use each technique, and understand when each is an option.